ACADÉMIE DES SCIENCES SOCIALES ET POLITIQUES
DE LA RÉPUBLIQUE SOCIALISTE DE ROUMANIE

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ÉDITIONS DE L'ACADÉMIE DE LA RÉPUBLIQUE SOCIALISTE DE ROUMANIE

To this purpose we shall analyse the matrix of reunion and inter- and section in the L_G lattice (see part I):

Matrix of Intersection

∩	a	b	c	d	e	f	g	h	j
a	a	d	c	d	h	f	g	h	o
b	d	b	g	d	e	o	g	h	j
c	c	g	c	g	o	f	g	h	o
d	d	d	g	d	h	o	g	h	o
e	h	e	o	h	e	o	o	o	j
f	f	o	f	o	o	e	o	o	o
g	g	g	g	g	o	o	g	o	o
h	h	h	o	h	h	o	o	h	o
j	o	j	o	o	j	o	o	o	j

Matrix of Reunion

∪	a	b	c	d	e	f	g	h	j
a	a	i	a	a	i	a	a	a	i
b	i	b	i	b	b	i	b	b	b
c	a	i	c	a	i	c	c	a	i
d	a	b	a	d	b	a	d	d	b
e	i	b	i	b	e	i	b	e	e
f	a	i	c	a	i	f	c	a	i
g	a	b	c	d	b	c	g	d	b
h	a	b	a	d	e	a	d	h	e
j	i	b	i	b	e	i	b	e	j

On the basis of these matrices we can build the complement table.

Complement Table in L_G

Objects	Complements
a	j
b	f
c	e, j
d	— —
e	c, f
f	b, f, j
g	— —
h	— —
j	a, c, f

Reading the matrix of the L_G lattice of Fig. 2/b which as we have seen describes, according to Fig. 2/a, the G type diagram from Fig. 1/c (see part I), we can notice that not all the elements of this lattice have got complements. The elements d ($A_1 - A_3$), g($A_2 - A_3$) and h($A_1 - A_2$) have no complement in this lattice. The elements a (A_1), b($E - A_3$) have only one complement, the elements c (A_2) and ($E - A_2$) have two complements and the elements f (A_3) and j ($E - A_1$) have three complements each (see the table). That means we cannot use these complements as negation since they are not functional. We shall introduce the notion of relative complement in a lattice.

"By a complement of u is meant any x of L satisfying the equations

$$u \cap x = 0 \tag{14}$$

$$u \cup x = i \tag{15}$$

By the commutativity of the meet and join operations, and by definition, the property of complementarity is symmetrical: if x is a complement of u, so is u a complement of x".[3]

[3] Gábor Szász, *Introduction to Lattice Theory*, Academic Press, New York and London, 1966, pp. 45, 46.

An interval bounded by two elements a and b so that $b \leqslant a$, and a and b belong to L, is said to be the set of all x elements, x belonging to L, for which $b \leqslant x \leqslant a$. We shall write this interval as $[a, b]$. A relative complement in an arbitrary lattice L is defined in relation to an interval. Let $[a, b]$ be such an interval in the lattice L and u an element of a, b. If an element x of the lattice L satisfies the equations:

$$u \cap x = a \tag{16}$$

$$u \cup x = b \tag{17}$$

we say that x is included in the sublattice $[a, b]$. Besides, it is a complement of u in this sublattice. On this basis, an element x in L, for which the relations (16) and (17) do exist is called a *relative complement of* u in $[a, b]$. The adjective *relative* points to the consideration of this complement as relative to sublattice. We can also say that x is the *relative complement of* u in relation to the pair of elements a and b.[4]

In fig. 1 an order by inclusions within the general affections is described.

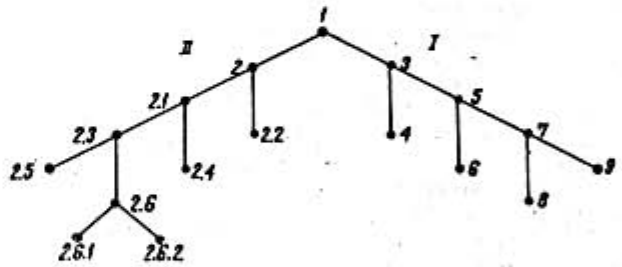

Fig. 1. — A partial diagram of general affections.

We have written internal affections (1), which are divided into complaints of the digestive system (2) and other internal affections, respectively, heart diseases (4), lung troubles (6), complaints of the renal system (8) and other complaints apart from the ones enumerated above. This can be considered a main lattice I, which describes the general classification of the internal affections. For each new chapter secondary lattices can be derived. In this way we have devised, in the above picture, the secondary lattice II, where the complaints of the digestive system are divided into stomach disorders (2. 1) and other complaints of the digestive system. Further on, the stomach disorders can be divided into gastritis (2.3) and other stomach disorders (2.4). In their turn, the gastrites can be divided into acute gastritis (2.5) and chronic gastritis (2.6). For chronic gastritis we shall consider two variants (2.6.1) and (2.6.2.).

[4] See also Gábor Szász, *op. cit.*, p.. 45, 46.

In fig. 1 we show a clarifying transformation of a polytomic division in a sequence of dichotomic divisions. It so happens in the interval [3, 9] as well as in the interval [2, 2.3]. In the interval [2.3, 2.6.2.] the division is "naturally" dichotomic.

The transposition of this classification or of a similar one, in a type G diagram can be operated by reducing the secondary lattice (II) to element (2). The other way round, starting from each of the element (4), (6), (8), (9), (2.2.), (2.4), (2.5.), a new tree lattice can be derived. These lattices can be of the second, the third or the n^{th} degree, depending on the number of branches, and we can re-read them according to the symbols we have used in fig. 1, by the number of the figure groups separated by full stops.

In order to get the inclusion ordering necessary to build a type G diagram and an L_G lattice, it is necessary to use a dichotomic order. For this we suggest the following definitions :

1. A lattice or an arbitrary interval in a lattice, where the ordering relation is inclusion, will be considered intensional if the ordering relation is of a genus-species type. That means that a defining feature of the genus is necessarily predicable for all the species of the division.

2. A lattice or an arbitrarily selected interval in a lattice, where the ordering relation is inclusion, will be considered an extensional lattice if the ordering relation is from whole to parts. By this we mean a relation in which the defining feature of the whole is not necessarily a predicate for a part of it, no matter which part, or that the collection which makes up the whole has no defining feature apart from the generic one of the whole, totality, set, or collection.

An example of intensional interval is the relation between the species "humans", the class "mammals" and the class "animals". An example of extensional relation can be the relation between the parts of an aeroplane and the aeroplane itself. Also, the example of a set, or a collection : the set of apples or the stamp collection, where the respective sets are different from the objects they include. In the case of an apple set, they are nothing else but a sum, without any other defining feature, apart from that of a set, a multitude, whereas apples are "fruit" and are part of the "fruit" genus.

The difference between an intensional interval and an extensional one is difficult to express in a mathematical pattern, because the reasoning way of the natural sciences cannot be free from any existential (ontologic) relation.

In the interval [2.3., 2.6.2] in fig. 1 there is such an intensional relation of the genus-species type. It is therefore an intensional interval. This relation cannot be disclosed unless we know the nature of the objects included in this interval. The main semilattice, beginning with object 3 in the same figure, is an extensional lattice because the element (3) does not describe a genus with certain features but a nosologic one (the same holds true about the elements (5) and (7)).

In other words, a polytomic division, as for instance the one of the internal diseases (1) can be rendered by a superior semilattice, like the main semilattice L of fig. 1 which is extensional because the objects (2), (4), (8) and (9) are species of the same division and the relations between

the objects (4) and (5) on the one hand and (3) on the other is similar to the one between parts and whole. The same about (6) and (7) on the one hand and (5) on the other, and (8) and (9) and (7). The relation between the objects (2), (3) and (1) is of the genus-species type, only (3) is an object made up of more species, it is therefore the complement of (2). This artifice makes it possible for us to order any classification, either "naturally" dichotomic or polytomic in a lattice of the L_G type. We shall consider it therefore as an instrument of dichotomic modelling of concepts in medical science.

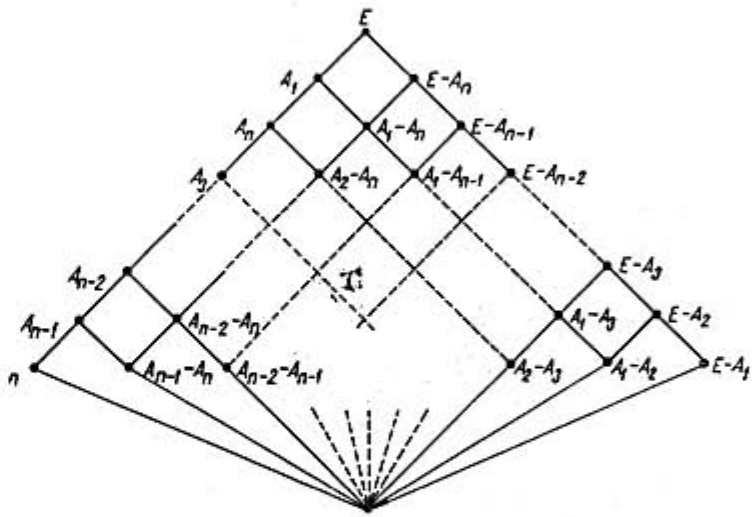

Fig. 2. — The diagram associated with an L_G type lattice with n objects.

In fig. 2 we have illustrated a method of building a diagram associated with an L_G type lattice of n objects. Such a diagram can be useful in organizing richer classifications.

This diagram can be used in order to select the interval where we wish to appraise truth of falsity of a medical assertion in the sense of a truth relative to a certain universe of discourse, which this time is limited to the selected interval.

We may try, with the help of this instrument to avoid a "relativization" of the notions of health and illness (as well as of those of Verum and Falsum) if we select intervals uniquely complementary (distributive). At the beginning we said that bivalent logic is too "strong", because any complement is unique in a Boole algebra. We have preferred the lattices which allow us to work with objects complemented in various ways. This is the relativization moment. However, to overcome this relativization without altering it, we have to come back and find the unique complement within a relativization of the complements. For this we need a device

of selecting a distributive sublattice or a distributive interval in a lattice. According to Birkhoff's criterion of distributivity, a lattice is distributive if, and only if, it has no sublattice isomorphic with either one of the lattices shown as Figs. 3/a and 3/b.

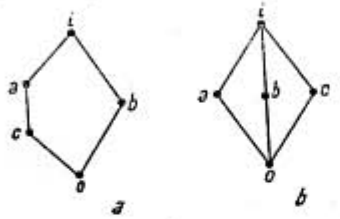

Fig. 3

Paraphrased: A lattice is distributive if and only if no interval [a, b] of the lattice includes an element having two different relative complements in [a, b].

COROLLARY I. Every element of a bounded distributive lattice has at most one complement.

COROLLARY 2. For the elements a, b, c of a distributive lattice $a \cap b = b \cap c$ and $a \cup b = b \cup c$ imply $a = b$.[5]

A G type lattice is not distributive generally, as we have seen so far. In fig. 4 there are some distributive lattices for which any interval has elements uniquely complemented (fig. 4 (b, c, d)).

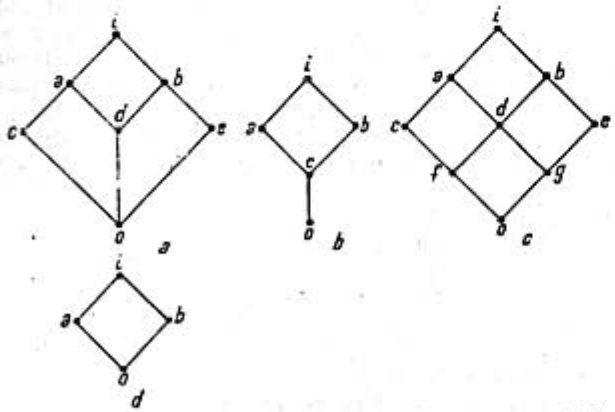

Fig. 4. — Examples of non-distributive and distributive lattices. Corresponding to Birkhoff's distributive criterion, in Fig. 4/a the element *e* has two different complements *a* and *c*; $a \cap e = o$, $c \cap e = o$, $a \cup e = i$, $c \cup e = i$. It is a non-distributive lattice. The lattices 4/b, 4/c and 4/d are distributives.

Thus we shall have the possibility to appraise the notion of health and illness in a universe of discourse (an interval) which sections a sector

[5] See Gábor Szász, *op. cit.* p. 90.

of a classification. If *a* is the complement of *b* and the reverse, in the diagram shown in fig. 4/d. we can replace these variables with some nosologic entities: *a* = acute gastrits, *b* = chronic gastritis, *i* = gastritis and *d* = 0. We can, on the occasion of seeing a patient *x* assert that "it is true that *x* suffers from acute gastritis but it is not true that he suffers from chronic gastritis" in case the diagnosis leads us to this assertion. Or, in another case we can say, about a patient *y* suffering from chronic gastritis with a minimum of symptoms, that "patient *y* is healthy as compared to patient *x*, of the former case, who suffers from acute gastritis". If we compare the health value of patient *y* to that of an individual *z*, who has no complaint medically disclosed, we shall be able to say that patient *y* is ill, as compared to person *z*.

Apart from the relativization of the notions of health and illness, as well as of those of *true* and *false*, the affirmation or negation of health or illness, we should like to call the rearder's attention upon the interpretation of negation in a type G lattice. It can be differently interpreted according to its being placed within an intensional or an extensional interval. Within an intensional interval negation is an element of hierarchy (ordering) which is a genus, denies the defining features of the genus and by this it also denies the predicable attribute of the species, a negation which is transferred to the species.

For instance, "patient *x* does not suffer from the Y complaint". If Y stands for gastritis and the general gastritis symptoms are absent, it is no longer necessary to see if the patient suffers from one form of gastritis or another.

In an extensional case when such generic features are not to be found, negation as an inclusive element of an interval in a lattice will have to be supported by the conjunction of the negation of the parts (the species) which make up this element, because there are no generic symptoms. Thus if patient *x* is in hospital for a digestive apparatus complaint, in order to find that other complaints are not present, we must minutely investigate each apparatus or system.

During the diagnosis whose stages we are not going to discuss now but we shall consider them as a sequence of plausible reasonings of a hypothetical deductive type, we need to affirm or deny, on the basis of a symptom (or of a group of symptoms) present or absent, a nosologic entity. For this we need a complementary couple of entities like health and illness. After selecting a hypothesis and supporting it we shall have to deny the complement of this hypothesis (nosologic entity). In order to get a correct exclusion we need a distributive lattice. We have seen that the lattices modifying pathological classifications are not of this type. Let us take as an example the classification of nephropathies according to Monasterio and Gianpolmo (fig. 5)[6].

[6] *Medicina Internă*, vol. V, Edit. Medicală, Bucureşti, 1957, p. 283.

In fig. 6 we have represented the dichotomic diagram of the same classification :

We can see that we have two lattices numbered I and II. The relations between element (1) and elements (2), (4), (6) and (7), of lattice I are of the genus-species type. All species are at the same intesional level.

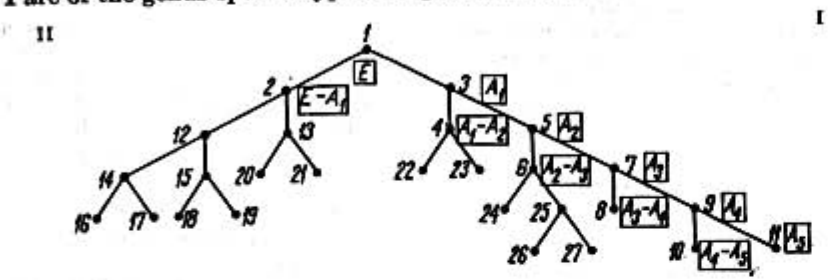

Fig. 5. — Nephropathies classification according to Monasterio and Gianpolmo (diagram).

1. Nephropathies; 2. Glomerularonephropathies; 3, 5, 9, 25. No distinct entity; 4. Tubal nephropathies (22. displastic and 23. tubal nephrosis); 6. Interstitial nephropathies; (24. suppurative, 26. no suppurative and 27. granulomatic); 7. Vascular nephropathies; 8. Nephroangiopathies; 10. Renal infarct; 11. Kidney stasis; 12. Glomerulonephritis (14 acute, 15 chronic); 13. Glomerulonephrosis (20. acute, 21. chronic); 14. Diffuse glomerulonephritis (16. acute, 17. chronic); 15. Focused glomerulonephritis (18. acute, 19. chronic).

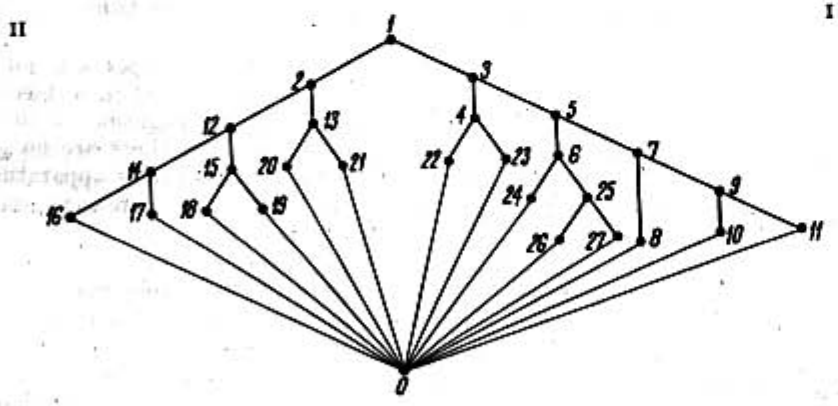

Fig. 6. — Lattice of nephropaties.

Because we wish to preserve the dichotomic structure we have to build the intermediary elements (3) and (5), which do not stand for intensional but for extensional levels. The structure of lattice II corresponds to a genus-species relation, an intensional relation. Lattice II can be reduced to element (2).

We shall take the case of a patient whose complaint started with fever (38°C), shivering, lumber pain, headache, nausea, vomiting and

with whom anurinary syndrome is also to be found (oliguria and microcorpus hematuria), a hydropigenious one (edemas, etc.) and whom we suspect to suffer from a nephropathy. We shall have to try and find a positive differential support for the diagnosis. The first hypothesis to be selected will be (α) glomerulonephropathy, (2). Further on we shall select the interval [2, 12, 13, 0] considering the affection as a (β) glomerulonephritis (12) and we shall exclude glomerulonephrosis (13) on the basis of the presence of hematuria at the start of the illness. We shall determine a new hypothesis (γ) diffuse glomerulonephritis, (14) and we shall select the interval [12, 14, 15, 0]. We shall exclude focused glomerulonephritis (15), on the contention that hematuria is of a microscopic type, there are edemas and a slight elevation of the blood pressure (16/10 cm Hg). The next hypothesis is (δ) diffuse glomerulonephritis — an acute form (16) with the interval [14, 16, 17] asserted on contention of the symptoms at the start and of the exclusion of the relative complement (17), the chronic form of evolution. We shall return to lattice I for the differential diagnosis. The complement of point (2) to which diagnosis (α) can be reduced is point (3) which includes the tubal, interstitial and vascular affections and their various species. For this we should exclude points (4) and (6) and (7) in turn.

We shall select a first species with the interval [1, 2, 4, 0] or [1, 16, 4, 0]. The tubal nephropathies (4) being generally divided in two groups : displastic nephropathies (22) and tubal nephroses (23), we shall take in turn the lattice intervals [1, 16, 22, 0] and [1, 16, 23, 0]. For the first interval, on the basis of an X-ray examination we shall deny congenital affections, (22), and from the second, we shall deny tubal nephrosis (23), in which the dominant symptoms are oliguria, hyperazothermy and cylindruria. Hematuria is associated only with chronic forms. Then we pass to interval [1, 16, 5, 0]. Again we shall select a smaller interval [1, 16, 6, 0] and we shall deny the three possible forms of interstitial nephropathy, one by one : the suppurating one (24) whose start is sudden but associated with piuria, [1, 16, 23, 0], the non-suppurating one (25) within [1, 16, 26, 0] when edemas are absent and hematuria is associated only to chronic forms as well as the granulomathy (27) in [1, 16, 27, 0]. The next lattice interval is [1, 16, 7, 0] where we can easily exclude nephroangiopathies (8) in the interval [1, 16, 8, 0] the renal infarct (10) in the interval [1, 16, 11, 0].

Besides the nephropathies taken as an initial hypothesis we can perform a differential diagnosis with other renal affections where hematuria is present by building lattices and selecting distributive intervals different from one another. So we can perform a differential diagnosis of renal lithiasis, of renal tuberculosis and renal cancer.

We can notice that every time we have selected a distributive interval of the initial lattice this allowed us to increase the degree of plausibility of the selected hypothesis by denying the complementary hypothesis. We have used a sequence of such negations each of them relative to the selected interval and the approach has not been considered completed before exhausting all the species of the genus that entered the classification. Each time we chose intermediary objects which allowed us to deny, one by one, the relative complements.

Of course, the same approach can be achieved also in lattices modelling classifications in a manner other than dichotomic. We think that in the manner suggested here we have succeeded in unifying an affirmation and excluding algoritm of complements on the basis of relative negation, which can make more efficacious and economic an automated diagnosis.

In the same classification of nephropathies, if the symptoms lead to a denial of the glomerular affection or at least of a glomerulo-nephritis (12) because it does not exhibit macroscopic or microscopic hematuria, it is no longer necessary to consider the entities diffuse (14) or focused (15) glomerulonephritis since they are denied by the absence of the dominant symptoms specific of the nephritis genus. By this we can reduce the number of steps in a diagnosis. We notice that the exclusion of a diagnosis can be rapidly performed in intensional intervals and with a much greater effort in extensional ones.

In this way we can understand why the physician, in front of a patient can establish a diagnosis in a very short time. He models his thinking, makes it relative, when necessary he can reduce it to a certain domain, a wide or a narrow one, a certain interval and at the same time he can preserve the total image — relating everything to the initial lattice. The use of negation in intensional intervals which is to be found in abundance in the classification of syndromes or pathogenic groups, can be an explanation of the physician's rapidity of thought when he establishes the diagnosis.

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